

Simulation of mold filling by a highly viscous fluid using the 2D indirect boundary element method

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Abstract

The results of simulation of mold filling by a highly viscous fluid are presented in this paper. The process of injection molding using a polymer melt is simulated. The mold has a rectangular shape and can have a coaxially placed rod. Evolving flow is characterized by a free surface undergoing significant deformations during the whole process. The indirect boundary element method for two-dimensional Stokes flows was used for calculations. Numerical procedures required for the boundary remeshing are described. Calculations were performed for both cases: top-down filling (gate at the top of the mold) and bottom-up filling (gate at the bottom of the mold). Differences in flow behavior are demonstrated for both variants. Comparison of material distribution in filled molds is performed. For this purpose interfaces between portions of fluid were marked and tracked up to the completion of filling process. The fountain flow regime is predominant for bottom-up mold filling. In the case of top-down mold filling, jet flow and film flow take place. Flow regimes leading to gas entrainments and welding which are considered unacceptable and cause product defects were shown.

Keywords: injection molding, boundary element method, free surface, filling, mold, polymer melt, material distribution, gas entrainment, weld lines, defects.

1 Introduction

Injection molding is one of the basic processes used in manufacturing products from polymer materials. There are optimum conditions providing required properties and high quality of products. Basic works in the field of injection molding [1–3] are dedicated to the problem of determining these conditions. Particularly, the process of mold filling by the polymer melt as well as methods of



simulation of this process are considered. It is noted that development of corresponding numerical methods is a very important issue. An example of such methods is boundary element method (BEM), basic provisions of which were formulated by Brebbia [4]. Efficiency of the method is determined by the fact that it considers the class of problems with moving boundaries. First of all BEM assumes calculation of boundary values of unknown functions and flow domain discretization is not necessary, e.g. BEM method was successfully used in [5, 6] for the purpose of study of fountain flow in the process of injection molding. This flow was studied in detail in a large number of works due to its important practical value for prediction of properties of polymer products [7]. Finite element method (FEM) is the most frequently used method regardless of obvious advantages of BEM. The main focus is set on determination of the shape of free surface and visualization of flow pattern which is especially important for manufacturing of multi-component products [8–10]. Component distribution visualization in these works is performed using FEM, solving pure convection equation and corresponding experiments. Experimental unit for visualization of free surface front movement is described in [11] and it is also shown that product defects are associated with this movement. Thus, material distribution in products is associated with the history of deformation of fluid elements in the process of mold filling. The purpose of this work is to obtain such material distributions and to show back history of their origin. It is assumed that mold filling process is performed for several polymer portions. These portions may have different properties (color, etc.) The basis of the approach applied is the use of indirect boundary element method (IBEM) and marker-particles separating different fluid portions continuously fed to the mold.

2 Problem formulation

Idealized process scheme consists in the following: portions of Newtonian fluid are sequentially and continuously fed to the mold through gate Γ_1 at specified flow rate, figure 1. This means that there is a vacuum in the mold or the latter has gas outlet openings. Gravity acceleration vector \mathbf{g} can be oriented both along the fluid flow (filling from the top) and in the opposite direction to the fluid flow (filling from the bottom). It is assumed that a planar problem statement is possible. Inlet boundary Γ_1 is located far enough from initial location of free surface Γ_3 to neglect its influence on the flow in the vicinity of Γ_3 . Velocity profile at Γ_1 corresponds to steady flow of viscous fluid in a 2D channel i.e. has a parabolic form. Inertial forces are considered negligible compared to the viscous forces, i.e. Reynolds number is low. Surface tension forces are neglected. As a rule these assumptions are true for injection molding of polymer materials [1–3]. No slip condition is specified for solid walls and traction vector components are considered equal to zero for free surface. Moreover, the free surface is subject to the kinematic condition.

Thus, taking into account conditions stated above the mathematical formulation of the problem is as follows. It is necessary to find a solution of Stokes equation



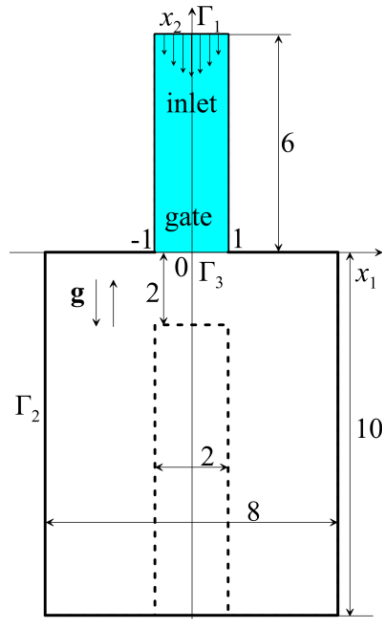


Figure 1: Diagram of mold with and without a central rod.

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad i, j = 1, 2 \quad (1)$$

and continuity equation

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

with boundary conditions at Γ_1 :

$$u_1 = 0, \quad u_2 = \frac{3}{2}(x_1^2 - 1), \quad (3)$$

at Γ_2 :

$$u_i = 0, \quad (4)$$

at Γ_3 :

$$t_i = St x_2 n_i, \quad (5)$$

where $\sigma_{ij} = -p_m \delta_{ij} + 2\dot{e}_{ij}$ – stress tensor components, $p_m = p - St x_2$ – modified pressure, p – pressure, $St = \frac{\rho(\mathbf{g} \cdot \mathbf{e}_2)L^2}{\mu U}$ – Stokes number, \mathbf{e}_2 – unit vector of x_2

axis, $\dot{e}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ – strain rate tensor components, $t_i = \sigma_{ij} n_j$ – traction vector components, n_j – components of outward-pointing normal vector to Γ_3 ,



L – gate half-width, U – average gate flow rate, μ – dynamic viscosity coefficient, ρ – density. Homogeneity of equation (1) is determined by the fact that the force of gravity is a potential force. Equations and boundary conditions (1)–(5) are written in dimensionless form. The following physical quantities are used as scales: length – L , velocity – U , pressure – $\mu U / L$.

After solving the boundary value problem (1)–(5) for known boundary Γ_3 , its new location is found from kinematic condition used in Lagrangian form:

$$\frac{dx_i}{dt} = u_i. \quad (6)$$

3 Numerical simulation

The IBEM is used for solving boundary value problem (1)–(5) for each time step. Stokes equation taking into account the gravity force is written in form of equation (1). This makes it possible to develop the solving method associated only with the discretization of flow domain boundary. The integral equations are as follows:

$$\begin{aligned} u_i(\mathbf{x}) &= \int_{\Gamma} G_{ij}(\mathbf{x}, \xi) \varphi_j(\xi) d\Gamma(\xi), \\ t_i(\mathbf{x}) &= \int_{\Gamma} F_{ij}(\mathbf{x}, \xi) \varphi_j(\xi) d\Gamma(\xi), \end{aligned} \quad (7)$$

where $\varphi_j(\xi)$ is the density of fictitious sources distributed along the boundary Γ of domain occupied by the fluid. The functions G_{ij} and F_{ij} are the fundamental singular solutions of the Stokes equation for velocity and tractions determined by formulae [12]:

$$G_{ij}(\mathbf{x}, \xi) = -\frac{1}{4\pi} \left(\delta_{ij} \ln \frac{1}{r} + \frac{y_i y_j}{r^2} \right), \quad F_{ij}(\mathbf{x}, \xi) = \frac{y_i y_j y_k n_k(\mathbf{x})}{\pi r^4},$$

where $y_i = x_i - \xi_i$, $r = (y_i y_i)^{1/2}$.

Constant elements are used. The discretized boundary integral equations in accordance to equations (7) acquire the form:

$$\begin{aligned} u_i(\mathbf{x}^p) &= \sum_{q=1}^N \varphi_j^q \Delta G_{ij}^{pq}, \\ t_i(\mathbf{x}^p) &= \sum_{q=1}^N \varphi_j^q \Delta F_{ij}^{pq}, \end{aligned} \quad (8)$$

where N – amount of the boundary elements, \mathbf{x}^p – the middle of element p (node p). In this case integrals of the components of fundamental traction $\Delta F_{ij}^{pq} = \int_{\Delta\Gamma^q} F_{ij}(\mathbf{x}^p, \xi) d\Gamma(\xi)$ and velocity $\Delta G_{ij}^{pq} = \int_{\Delta\Gamma^q} G_{ij}(\mathbf{x}^p, \xi) d\Gamma(\xi)$ tensors are

calculated analytically including the elements with singularities ($p = q$). All relations are given in [6]. The advantages of use of constant elements are as follows: known singularity at the three-phase contact line [13] is neglected;

approximate numerical integration can be avoided; efficient algorithm for free surface movement with large deformations can be developed.

Finite difference Euler scheme is used for free surface movement in accordance with (6). Element edges where velocity values are calculated are moved. Time step is calculated using the Courant condition.

Interfaces are specified between each of 10 fluid portions fed to the mold. Marker sequence is set at the gate. Then markers move in accordance with calculated velocity field and kinematic condition (6). New markers are added in sequence where the distance between adjacent ones grows significantly. Thus the interface tracking process is realized.

The boundary mesh refinement algorithm for free surface front movement includes the following procedures:

1. Adding elements: performed by means of division of an element having a length greater than specified value.
2. Removing elements by merging adjacent ones. Performed in order to avoid accumulation of edges at drain areas. Element having a length less than specified value is merged with the shortest adjacent element.
3. Welding. In the areas of potential welding the slope angle of boundary elements relative to each other decreases and becomes less than a specified value, nodes move towards each other. Thus, they tend to stick together. The common edge in this case moves to a new location – equidistant from the two adjacent ones. A marker is set at edge's previous location and its movement is tracked in the process of simulation. Thus, the impingement of a free surface on itself is simulated and weld lines are tracked.

Results were obtained for initial number of boundary elements equal to 200. In the process of calculation the number of elements reached 800. Efficient numerical algorithm [6] for matrix generation and solving the system of linear equations (8) was used.

4 Results and discussion

Simulation results of filling of mold without a central rod for fluid fed to the mold in the direction of gravity force (top-down, $St < 0$) are shown in figure 2. Flow character is determined by jet stability [14]. When the jet buckles the symmetry of distribution of mass of portions is lost and weld lines are formed ($St = -0.5$, figure 2). These lines are marked by a sequence of white markers. Such filling regimes are usually undesired as they decrease strength characteristics of the product [2]. The presence of a central rod, figure 3, leads to stabilization of the jet from the gate and the mold is filled by the film of fluid draining down the rod. For small values of Stokes number significant voids are formed in the vicinity of corners of the central rod and upper part of the mold near the gate is filled first ($St = -0.025$, figure 3). It should be noted that it is assumed that there is no gas in the mold (vacuum). Otherwise cavities will be formed in case if there are no openings in corresponding areas.



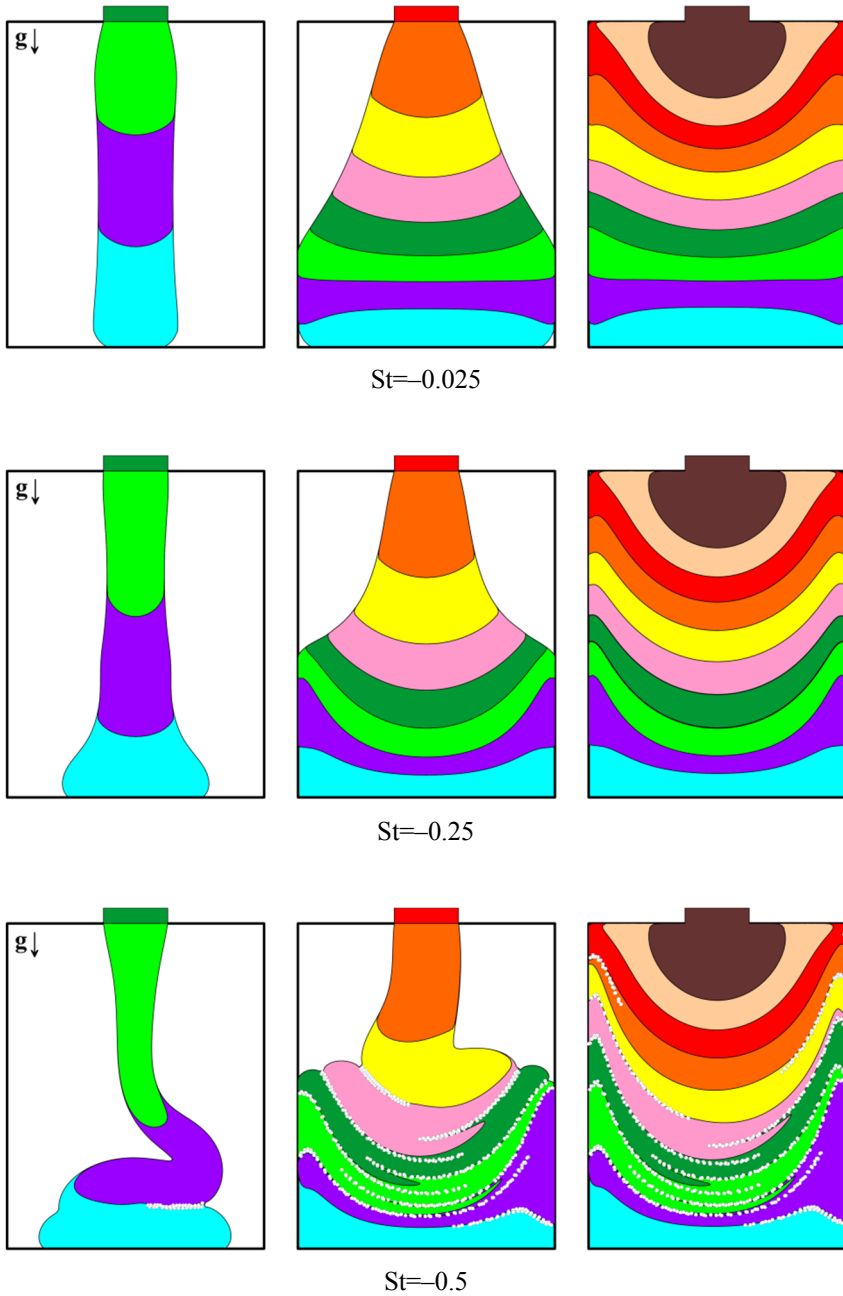


Figure 2: Mold filling in the direction of gravity force (30%, 70%, 100% fill).

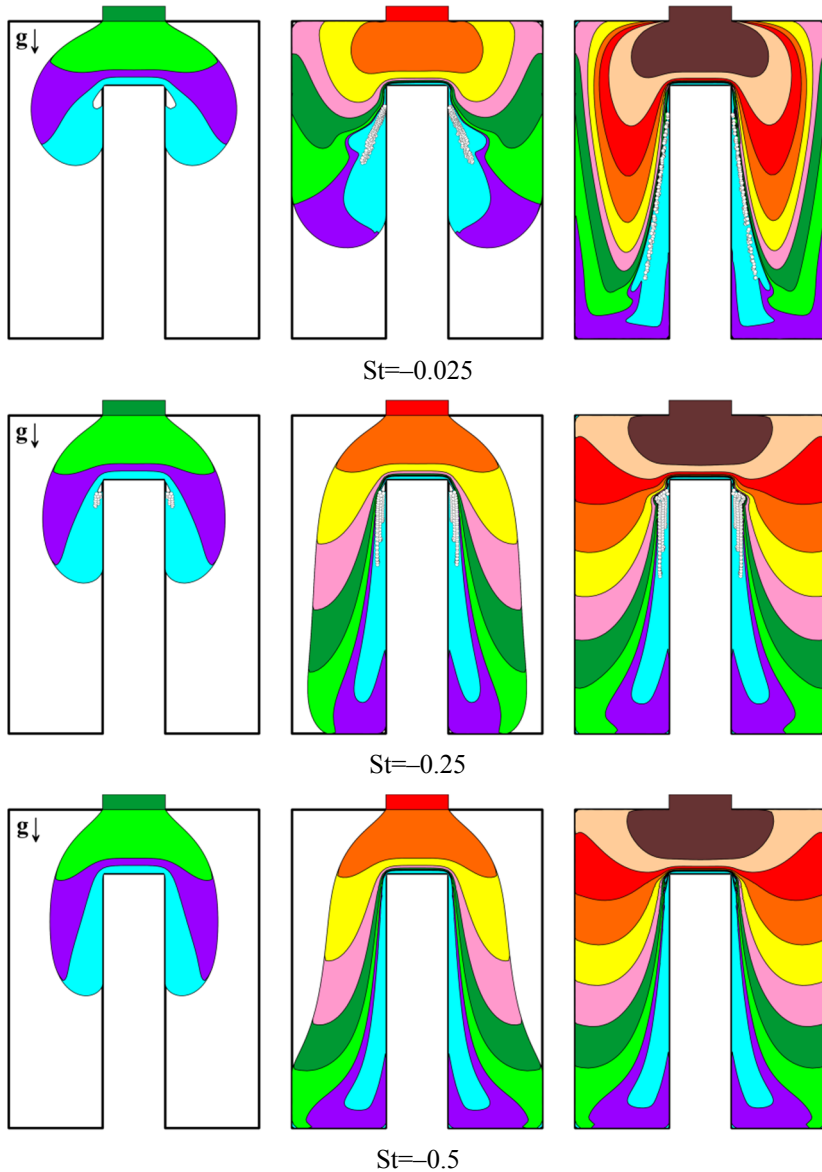


Figure 3: Filling of the mold with a rod in the direction of gravity force (30%, 70%, 100% fill).

Figures 4 and 5 show mold filling regimes for injection molding process when the fluid is supplied in the direction opposite to gravity force (bottom-up filling, $St > 0$). Series of calculations represented in figures 2–5 for the same absolute values of Stokes number makes it possible to compare flow patterns and material distribution in a filled mold for both (top-down and bottom-up) filling options. It

should be noted that for small values of Stokes number the character of bottom-up mold filling ($St=0.025$, figures 4 and 5) is practically the similar to the top-down mold filling ($St=-0.025$, figures 2 and 3). These cases are close to that when the gravity force is absent ($St=0$). The difference becomes more significant after $|St|\sim 0.05$.

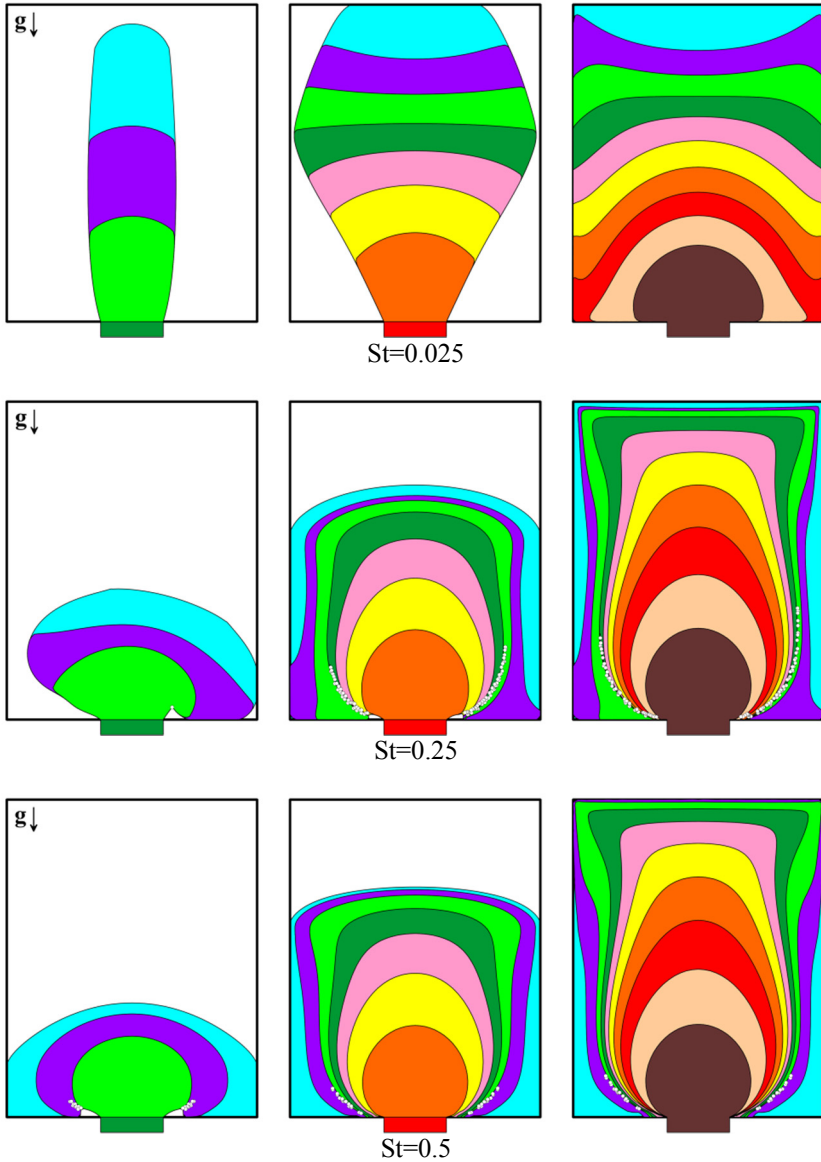


Figure 4: Mold filling in the direction opposite to gravity force (30%, 70%, 100% fill).

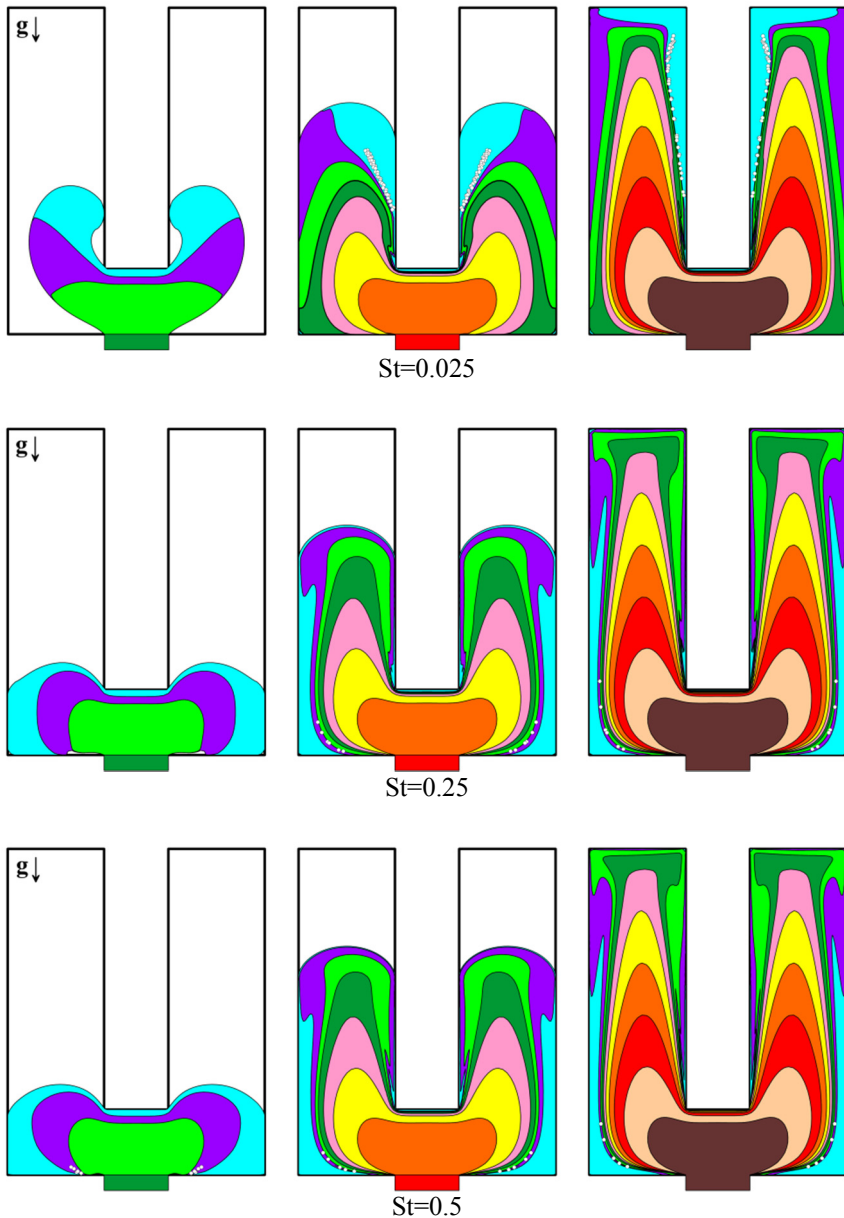


Figure 5: Filling of the mold with a rod in the direction opposite to gravity force (30%, 70%, 100% fill).

Depending on the value of Stokes number at the initial stage the mold is filled in jet regime ($St=0.025$, figure 4) or spreading regime ($St\sim 1$ and higher). For the case without a central rod, figure 4, the flow character in the mold is determined

by the gate outflow regime. There is also a transient regime which is characterized by jet deformation under the influence of gravity which results in jet either losing stability and collapsing to the side losing symmetry ($St=0.25$, figure 4), or taking a mushroom-like shape returning to the side and spreading further ($St=0.5$, figure 4). This is accompanied by formation of voids which are classified in finished products as defects. These voids are located in the vicinity of the gate as it is pointed out in paper [15] where gas entrainment flow regimes were investigated. As the mold in the considered case is not filled with gas (vacuum) weld lines (indicated by white markers) are formed in these locations instead of voids.

As calculations indicated the spreading regime when there are no voids registered in the vicinity of the gate corners takes place at $St \geq 10$. However, the sizes of voids at $St \sim 2$ and higher become insignificant. It should be noted that the presence of such flow character was pointed out in [15] in the shape casting process simulation for techniques where inertial effects and surface tension were taking into account. This research indicates that such defects are formed in the injection molding process also when simulation is based on the creeping flow approximation. Material distribution pattern significantly depends on the regime implemented. It is especially true for the location of the first portions.

The presence of the rod makes the flow pattern significantly more sophisticated, figure 5. Thus, for small values of Stokes number at the initial stage the jet has a form of a bowl during its splitting by the rod ($St=0.025$, figure 5). Then it either continues to creep up the rod if the value of Stokes number is small ($St=0.025$, figure 5) or returns to the outer wall or the bottom of the mold taking a very sophisticated shape for greater values of Stokes number ($St=0.25$, $St=0.5$, figure 5). As the value of Stokes number increases the gap between the central rod and vertical walls fills up analogous to the filling of a vertical channel (fountain flow). V-shaped interfaces which can be seen were also observed in many works [2]. In this case weld lines are located either along the rod starting from its corners ($St=0.025$, figure 5) or in the gate vicinity ($St=0.25$, $St=0.5$, figure 5).

As one can see from figures 2–5, interpenetration of the portions of fluid becomes more significant as the values of Stokes number increase. Thereby the mixing effect intensifies. This leads to the fact that at large Stokes numbers it is difficult to distinguish interfaces between the portions clearly.

5 Conclusions

The results presented indicate that IBEM method is an efficient tool for analysis of mold filling by means of injection molding. This process is accompanied by evolution of free surface which may lead to defects in finished products. These defects are associated with possible formation of gas entrainments and weld lines. Even a monolithic product may contain surfaces (weld lines) – sources of internal stresses which affects strength characteristics of the product. It is indicated that special focus is to be placed on the vicinity of the mold gate and corner points of the central rod. Two basic mold filling regimes are possible: jet and fluid spreading down the solid walls. In the first case when jet stability is lost weld lines are usually

formed. As for the second case the main possible cause of defect formation is the formation of cavities in the process of front deformation in the vicinity of corners. Material distributions inside the product are obtained for different portions. These distributions indicate the presence of V-shaped interfaces typical for fountain flow [2]. Such interfaces are typical for first portions in the process of bottom-up mold filling. At the same time it should be noted that there are a lot of various material distribution patterns obtained. In all considered cases there are individual ranges of Stokes number values providing a favorable flow regarding the product quality: without internal defects and weld lines. The primary objective of design of production equipment is to determine these ranges.

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